OCR Maths FP1

Topic Questions from Papers

Matrices

Answers

1	(i) $A^2 = \begin{pmatrix} 3 & 8 \\ 4 & 11 \end{pmatrix}$	M1 A1		Attempt to find A ² , 2 elements correct All elements correct
	$4A = \begin{pmatrix} 4 & 8 \\ 4 & 12 \end{pmatrix}$	M1	4	Use correct matrix 4A
	(ii) $\mathbf{A}^{-1} = 4\mathbf{I} - \mathbf{A}$	A1 M1 A1	426	Obtain given answer correctly

(Q2, June 2005)

2 (i)	det(B) = 0			
	3a - 4 + 6 = 0	M1		Attempt to solve their det(B)=0
	$a = \frac{-2}{3}$	A1 A1	3	Obtain given answer
(ii)		M1		Show correct process for adjoint entries
	$\frac{1}{3a+2} \begin{pmatrix} 3 & 1 & -4 \\ -4 & 2a & a+6 \\ 2 & -a & a-2 \end{pmatrix}$	A1		Obtain at least 4 correct entries in adjoint
	$3a+2$ $\begin{pmatrix} 2 & -a & a-2 \end{pmatrix}$	B1 A1	4	Divide by their determinant Obtain completely correct answer
(iii)	a = -1	M1		
	$\begin{pmatrix} -11\\17\\-9 \end{pmatrix}$	M1		Attempt product of B-1C
	x = -11, y = 17, z = -9	A1		Obtain correct answer
			3	

(Q7, June 2005)

3	(i) $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$	B1B1	2	Each column correct
	(ii) Shear, e.g. (0,1) transforms to (3,1)	B1B1	2	One example or sensible explanation
	$\begin{bmatrix} \text{(iii)} & \mathbf{M} = \begin{pmatrix} 2 & 3 \\ 0 & 1 \end{pmatrix} \end{bmatrix}$	M1 A1	2	Attempt to find DC (not CD) Obtain given answer

(Q9, June 2005)

4	$2\begin{bmatrix} 21\\13 \end{bmatrix} - 1\begin{bmatrix} 11\\13 \end{bmatrix} + 3\begin{bmatrix} 12\\11 \end{bmatrix}$	M1		Show correct expansion process, allow sign slips
	2 x 5 - 1 x 2 +3 x -1 5 (ii)	A1 A1 B1ft	3 1 4	Obtain correct (unsimplified) expression Obtain correct answer State that M is non-singular as det M non-zero, ft their determinant

(Q3, Jan 2006)

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$				1	T
State or imply $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$ Use this result and obtain $\mathbf{B}^{-1} = \mathbf{C}^{-1}\mathbf{A}$, or equivalent matrix algebra M1 Afft M1 Afft M2 M3 M4 M4 M5 M4 M1 Afft M1 Afft M1 Afft M1 Afft M2 M3 M4 M4 M4 M5 M6 M7 M7 M8 M7 M7 M8 M8 M7 M8 M8 M8 M8 M8 Solve one pair of simultaneous equations Each pair of answers Correct \mathbf{B}^{-1}	5			2	diagonal
Or					Use this result and obtain $\mathbf{B}^{-1} = \mathbf{C}^{-1}\mathbf{A}$, or
$\mathbf{B} = \mathbf{A}^{-1} \mathbf{C}$ $\mathbf{B} = \frac{1}{5} \begin{pmatrix} 0 & -2 \\ 5 & 14 \end{pmatrix}$ $\mathbf{M}1$				5	pair
$ \frac{1}{2} \begin{pmatrix} 14 & 2 \\ -50 \end{pmatrix} $ Or $ AB = \begin{pmatrix} 2a + c & 2b + d \\ a + & 3c & b + & 3d \end{pmatrix} $ $ a = 0, c = 1, b = -0.4, d = 2.8 $ $ \frac{1}{2} \begin{pmatrix} 14 & 2 \\ -50 \end{pmatrix} $ Al $ A1 \qquad Matrix multn. Two elements correct All elements correct All elements correct A1 Solve one pair of simultaneous equations Each pair of answers Correct B-1$		` '	B1		Find A ⁻¹
Or $AB = \begin{pmatrix} 2a + c & 2b + d \\ a + & 3c & b + & 3d \end{pmatrix}$ $a = 0, c = 1, b = -0.4, d = 2.8$ $\frac{1}{2} \begin{pmatrix} 14 & 2 \\ -50 \end{pmatrix}$ All elements correct Correct \mathbf{B}^{-1} Find \mathbf{AB} Solve one pair of simultaneous equations Each pair of answers Correct \mathbf{B}^{-1}		$\mathbf{B} = \frac{1}{5} \begin{pmatrix} 0 & -2 \\ 5 & 14 \end{pmatrix}$	M1		Premultiply by A ⁻¹ stated or implied
Or $AB = \begin{pmatrix} 2a + c \ 2b + d \\ a + 3c \ b + 3d \end{pmatrix}$ $a = 0, c = 1, b = -0.4, d = 2.8$ $\frac{1}{2} \begin{pmatrix} 14 \ 2 \\ -50 \end{pmatrix}$ B1 B1 M1 A1A1 A1 B1 Solve one pair of simultaneous equations Each pair of answers Correct B ⁻¹		$\frac{1}{2} \begin{pmatrix} 14 & 2 \\ -50 \end{pmatrix}$			
$a = 0, c = 1, b = -0.4, d = 2.8$ $\frac{1}{2} \begin{pmatrix} 14 & 2 \\ -50 \end{pmatrix}$ $A1A1$ $A1$ Solve one pair of simultaneous equations $Each pair of answers$ $Correct B-1$			AI		Correct B ⁻¹
$\begin{bmatrix} \frac{1}{2} & 14 & 2 \\ -5 & 0 \end{bmatrix}$ A1A1 $\begin{bmatrix} \text{A1A1} \\ \text{A1} \end{bmatrix}$ Solve one pair of simultaneous equations} $\begin{bmatrix} \text{Each pair of answers} \\ \text{Correct } \mathbf{B}^{-1} \end{bmatrix}$					Find AB
Correct B ⁻¹					·
			A1	7	

(Q6, Jan 2006)

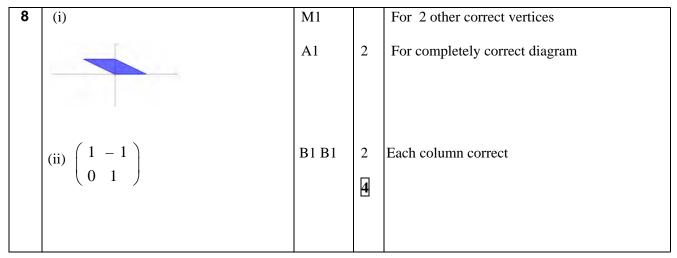
6	(i) $\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} \begin{pmatrix} 2 \\ -2 \end{pmatrix} \begin{pmatrix} 0 \\ -2 \end{pmatrix}$	B1 B1 B1	3	For correct vertex (2, -2) For all vertices correct For correct diagram
	(ii) Either $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	B1,B1 B1		Reflection, in <i>x</i> -axis Correct matrix
	$\left(\begin{array}{cc}2&0\\0&2\end{array}\right)$	B1,B1 B1	6	Enlargement, centre O s.f.2 Correct matrix
	Or $\begin{pmatrix} -10\\0&1 \end{pmatrix}$	B1,B1 B1		Reflection, in the <i>y</i> -axis Correct matrix
	$\left(\begin{array}{cc} -2 & 0 \\ 0 & -2 \end{array}\right)$	B1,B1 B1		Enlargement, centre O s.f. –2 Correct matrix
	Or $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$	B1,B1 B1		Stretch, in <i>x</i> -direction s.f. 2 Correct matrix
	$\left(\begin{array}{cc} 1 & 0 \\ 0 & -2 \end{array}\right)$	B1,B1 B1		Stretch, in <i>y</i> -direction s.f2 Correct matrix
			9	

(Q8, Jan 2006)

7		B1		Two elements correct
	i) $\begin{pmatrix} 7 & 4 \\ 0 & -1 \end{pmatrix}$	B1	2	All four elements correct
	(ii) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$	B1		$\mathbf{A} - \mathbf{B}$ correctly found
	<i>k</i> = 3	B1	2	Find k
			4	

(Q1, June 2006)

t



(Q2, June 2006)

3

9	(i)	M1		Attempt at matrix multiplication	
	$\mathbf{A}^2 = \begin{pmatrix} 4 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{A}^3 = \begin{pmatrix} 8 & 0 \\ 0 & 1 \end{pmatrix}$	A1 A1	3	Correct A ² Correct A ³	

(Q7, June 2006)

10	(i)	M1		Correct expansion process shown
	$a \begin{bmatrix} a & 0 \\ 2 & 1 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} + 2 \begin{bmatrix} 1 & a \\ 1 & 2 \end{bmatrix}$	A1		Obtain correct unsimplified expression
	a^2-2a	A1	3	Obtain correct answer
	(ii)	M1		Solve their det $\mathbf{M} = 0$
	a = 0 or a = 2	A1A1ft	3	Obtain correct answers
	(iii) (a)	B1 B1		Solution, as inverse matrix exists or \mathbf{M} non- singular or $\det \mathbf{M} \neq 0$
	(b)	B1 B1	4	Solutions, eqn. 1 is multiple of eqn 3
			10	

(Q8, June 2006)

11	(i) $a = -3$	B1	1	State correct value
	(ii) $2a - 3 = 7$ or $3a - 6 = 9$	M1		Sensible attempt at multiplication
	a = 5	A1	2	Obtain correct answer
			3	

(Q1, Jan 2007)

12		M1		For at least two correct images
	$ (i) \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix} \begin{pmatrix} 3 \\ 0 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \end{pmatrix} $	A1	2	For correct diagram, co-ords.clearly written down
	(ii) 90 ⁰ clockwise, centre origin	B1 B1		Or equivalent correct description
	$\left(\begin{array}{c} 0 & 1 \\ -1 & 0 \end{array}\right)$	B1	3	Correct matrix, not in trig form
	(iii) Stretch parallel to <i>x</i> -axis, s.f. 3	B1 B1		Or equivalent correct description, but must be a stretch for 2 nd B1
	$\left(\begin{array}{c} 3\ 0 \\ 0\ 1 \end{array}\right)$	B1 B1	4 9	Each correct column

(Q9, Jan 2007)

		1	ı	
13	(i)	M1		Show correct expansion process for
		M1		3 x 3
	$\Delta = \det \mathbf{D} = 3a - 6$	A1		Correct evaluation of any 2 x 2 det
		M1		Obtain correct answer
		A1		Show correct process for adjoint
		B1		entries
	$\mathbf{D}^{-1} = \frac{1}{\Delta} \begin{pmatrix} 3 - 2 & 4 \\ -3 & a - 2a \\ -3 & a \ a - 6 \end{pmatrix}$	A1	7	Obtain at least 4 correct entries in
				adjoint
	(ii) $\frac{1}{\Delta}$ $\begin{pmatrix} 5\\2a-9\\5a-15 \end{pmatrix}$	M1		Divide by their determinant
	$\left(5a-15\right)$	A1A1A1 ft all 3	4	Obtain completely correct answer
			11	
				Attempt product of form D ⁻¹ C , or eliminate to get 2 equations and solve Obtain correct answers, ft their inverse

(Q10, Jan 2007)

14	(i) $\frac{1}{2}$ $\begin{pmatrix} 5 & -1 \\ -3 & 1 \end{pmatrix}$	B1 B1	2	Transpose leading diagonal and negate other diagonal or solve sim. eqns. to get 1 st column Divide by the determinant or solve 2 nd pair to get 2 nd column
	(ii) $\frac{1}{2} \begin{pmatrix} 2 & 0 \\ 23 & -5 \end{pmatrix}$	M1 M1(indep) A1ft A1ft	4 6	Attempt to use B ⁻¹ A ⁻¹ or find B Attempt at matrix multiplication One element correct, a.e.f, All elements correct, a.e.f. NB ft consistent with their (i)

(Q4, June 2007)

15	(i)	M1 M1		Show correct expansion process Show evaluation of a 2 x 2
	a(a-12) + 32	A1	3	determinant
	(ii)			Obtain correct answer a.e.f.
	$\det \mathbf{M} = 12$	M1	2	
	non-singular	A1ft		Substitute $a = 2$ in their determinant
	(iii) EITHER	B1		
	OR	M1		Obtain correct answer and state a consistent conclusion
	ON	A1	3	consistent conclusion
		M1 A1		det M = 0 so non-unique solutions
		A1		Attempt to solve and obtain 2
				inconsistent equations
				Deduce that there are no solutions
				Substitute $a = 4$ and attempt to solve Obtain 2 correct inconsistent equations
			8	Deduce no solutions

(Q7, June 2007)

16	(i) $\begin{pmatrix} \sqrt{2} & 0 \\ 0 & \sqrt{2} \end{pmatrix}$	B1	1	Correct matrix
	(ii) Rotation (centre <i>O</i>), 45 ⁰ , clockwise (iii)	B1B1B1	3	Sensible alternatives OK, must be a single transformation
		B1	1	Matrix multiplication or combination of transformations
	(iv) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$	M1 A1	2	For at least two correct images For correct diagram
	(v) $\det \mathbf{C} = 2$	B1		State correct value
	area of square has been doubled	B1	2	State correct relation a.e.f.
			9	

(Q9, June 2007)

 \boldsymbol{x}

17 (i) 1 1	M1	For 2 other correct vertices seen, condirection of shear seen	rect
(1, -1)	A1	2 For completely correct diagram, muscales	st include
$ \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} $	B1 B1	$ \begin{array}{c cccc} 2 & & & & & \pm \\ 4 & & & & & \pm \\ \text{Each column correct} \end{array} $	

(Q1, Jan 2008)

$ \begin{array}{c c} $	B1 B1	2	4 B seen or implied or 2 elements correct Obtain correct answer
(ii) $ \begin{pmatrix} 8 & 16 & -4 \\ 0 & 0 & 0 \\ 6 & 12 & -3 \end{pmatrix} $	M1 A1A1A1	4	Obtain a 3 x 3 matrix Each row (or column) correct
(iii) (8)	M1 A1	2 8	Obtain a single value Obtain correct answer, must have matrix

(Q5, Jan 2008)

19	(i)		M1		Use det $\mathbf{A} = 0$	
		a = -6	A1	2	Obtain correct answer	
	(ii)	$\mathbf{A}^{-1} = \frac{1}{a+6} \begin{pmatrix} 1 & -3 \\ 2 & a \end{pmatrix}$	B1 B1ft		Both diagonals correct Divide by det A	
		$x = \frac{4}{a+6}$, $y = \frac{2-a}{a+6}$	M1		Premultiply column by A^{-1} , no other method Obtain correct answers from their A^{-1}	
			A1ft			
			A1ft	5		
				7		

(Q7, Jan 2008)

20 (i)	$\begin{pmatrix} 1 & 1 \\ 5 & -1 \end{pmatrix}$	B1	Two elements correct
		B1	All four elements correct
(ii)	EITHER	B1	Both diagonals correct
	$\frac{1}{3}\begin{pmatrix} 2 & -1 \\ -5 & 4 \end{pmatrix}$	B1	Divide by determinant
	α β $\alpha\beta$	2	
	O.A.	B1 B1	Solve sim. eqns. 1 st column correct 2 nd column correct

(Q1, June 2008)

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1 (i)	B1 Enlargement (centre O) scale factor 6
	1
(ii)	B1 Reflection
	B1 Mirror line is $y = x$
	2
(iii)	B1 Stretch in y direction
	B1 Scale factor 6, must be a stretch
	2
(iv)	B1 Rotation
	B1 36.9° clockwise or equivalent
	2
	(Q7. June 2

(Q7, June 2008)

22 (i)	M1 A1 2	Find value of det AB Correct value 2 seen
(ii)	M1 A1 B1	Show correct process for adjoint entries Obtain at least 4 correct entries in adjoint Divide by their determinant
$ (\mathbf{AB})^{-1} = \frac{1}{2} \begin{pmatrix} 0 & 3 & -1 \\ 0 & -1 & 1 \\ 2 & 6 - 3a & a - 6 \end{pmatrix} $	A1	Obtain completely correct answer
	4	
(iii) EITHER	<u></u>	State or imply $(\mathbf{A}\mathbf{B})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$
	A1	Obtain $\mathbf{B}^{-1} = (\mathbf{A}\mathbf{B})^{-1} \times \mathbf{A}$
	M1	Correct multiplication process seen
	A1	Obtain three correct elements
$\mathbf{B}^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 2 \\ -6 & 2 & -2 \end{pmatrix}$	A1	All elements correct
OR	5 M1 A1 M1 A1	Attempt to find elements of B All correct Correct process for B ⁻¹ 3 elements correct All elements correct
		(Q10, June 2008)

23	$\begin{array}{ccc} \mathbf{(i)} & \frac{1}{10} \begin{pmatrix} 5 & 0 \\ -a & 2 \end{pmatrix} \end{array}$	B1 B1	2	Both diagonals correct Divide by correct determinant	
	(ii) $\begin{pmatrix} 3 & -2 \\ 2a & 6 \end{pmatrix}$	B1 B1	2 4	Two elements correct Remaining elements correct	

(Q2, Jan 2009)

24		B1		State or use correct result
		M1		Combine matrix and its inverse
	$(0 \ 0)$	A1		Obtain I or I^2 but not 1
		A1	4	Obtain zero matrix but not 0
	(0 0)		4	S.C. If $0/4$, B1 for $AA^{-1} = I$

(Q4, Jan 2009) ×

25	Either	M1		Consider determinant of coefficients of LHS
		M1		Sensible attempt at evaluating any 3×3 det
	4k - 4	A1		Obtain correct answer a.e.f. unsimplified
		M1		Equate det to 0
	k = 1	A1ft	5	Obtain $k = 1$, ft provided all M's awarded
	Or	M1		Eliminate either x or y
		A1		Obtain correct equation
		M1		Eliminate 2 nd variable
		A1		Obtain correct linear equation
		A1		Deduce that $k = 1$
			5	

(Q5, Jan 2009)

	$ \begin{pmatrix} \mathbf{(iii)} & \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \\ \mathbf{(iv)} \end{pmatrix} $	B1 B1 B1B1B1	2 3 9	Each column correct Rotation, 90°, clockwise about <i>O</i> S.C. If (iii) incorrect, B1 for identifying
	(ii) (0 1)	B1 DB1	2	Reflection, in $y = -x$
26	(i) Either Or	B1 DB1 B1 DB1	2	Reflection, in <i>x</i> -axis Stretch parallel to <i>y</i> -axis, s.f. –1

(Q6, Jan 2009)

27	$3a + 5b = 1, \ a + 2b = 1$	M1		Obtain a pair of simultaneous
		M1		equations
	a = -3, b = 2	A1 A1	4	Attempt to solve
			4	Obtain correct answers.

(Q2, June 2009)

28	(i)	B1		Find coordinates (0, 0) (3, 1) (2, 1)
		B1		(5, 2) found
		B1	3	Accurate diagram sketched
	(ii) $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	B1 B1	2	Each column correct
	(iii) Either	B1		Correct inverse for their (ii) stated
	$\begin{pmatrix} 1 & 2 \end{pmatrix}$	M1		Post multiply C by inverse of (ii)
	0 1			
	$\begin{pmatrix} 0 & 1 \end{pmatrix}$	A1ft		Correct answer found
	Or	M1		Set up 4 equations for elements from
				correct matrix multiplication
		A2ft		All elements correct, -1 each error
		B1		Shear,
		B1		x axis invariant or parallel to x-axis
		B1	6	eg image of (1, 1) is (3, 1)
			11	SR allow s.f. 2 or shearing angle of
				correct angle to appropriate axis

(Q8, June 2009)

29	(i) $a \begin{vmatrix} a & 1 \\ - \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & a \\ 1 & 1 \end{vmatrix}$	M1 A1		Correct expansion process shown Obtain correct unsimplified
		711		expression
	$2a^2-2a$	A1	3	
				Obtain correct answer
	(ii)	M1		
	a = 0 or 1	A1ft		Equate their det to 0
		A1ft	3	Obtain correct answers, ft solving a quadratic
	(iii) (a)	B1 B1		Equations consistent, but non unique solutions
	(b)	B1		Correct equations seen &
		B1	4	inconsistent, no solutions
			10	

(Q9, June 2009)

30 (i) $\begin{pmatrix} a-4 & 2 \\ 3 & 0 \end{pmatrix}$	B1 B1	2	Two elements correct Remaining elements correct
(ii) $4a - 6$ $a = \frac{3}{2}$	B1 M1 A1	3	Correct determinant Equate det A to 0 and solve Obtain correct answer a. e. f.

(Q1, Jan 2010)

31 (i)		B1 B1	2	Rotation 90° (about origin) Anticlockwise
(ii)	Either	M1		Show image of unit square after reflection in $y = -x$
	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	A 1		Deduce reflection in <i>x</i> -axis
	Or	B1ft B1ft M1 A1 B1B2	1	Each column correct ft for matrix of their transformation Post multiply by correct reflection matrix Obtain correct answer State reflection, in <i>x</i> -axis C. If pre-multiplication, M0 but B1 B1 Available for correct description of their matrix
				(Q5, Jan 2010)
32 (i)		M1		Show correct expansion process for
		M1		3×3 or multiply adjoint by A Correct evaluation of any 2×2 at any stage
	$\det \mathbf{A} = \Delta = 6a - 6$	A1		Obtain correct answer
	$\mathbf{A}^{-1} = \frac{1}{\Delta} \begin{pmatrix} 3a - 1 & a + 1 & -4 \\ 1 & 2a - 1 & -2 \\ -3 & -3 & 6 \end{pmatrix}$	M1		Show correct process for adjoint entries
		A1		Obtain at least 4 correct entries in adjoint
		B1 A1	7	Divide by their determinant Obtain completely correct answer
(ii)	$\frac{1}{\Delta} \begin{pmatrix} 5a-7\\4a-5\\3 \end{pmatrix}$	M1		Attempt product of form A ⁻¹ C or eliminate to get 2 equations and solve
	(3)	A1A ft all	3	Obtain correct answer S.C. if det now omitted, allow max A2 ft
		11		

(Q9, Jan 2010)

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33	(i)	(-7)
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- M1 Obtain a single value
- A1 2 Obtain correct answer as a matrix

 $\mathbf{BA} = \begin{pmatrix} 5 & -20 \\ 3 & -12 \end{pmatrix}$

$$\begin{pmatrix} -7 & -20 \\ 11 & -20 \end{pmatrix}$$

- M1 Obtain a 2×2 matrix
- All elements correct
- B1 4C seen or implied by correct answer
- B1ft 4 Obtain correct answer, ft for a slip in **BA**

6

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(Q2, June 2010)

34 (a)
$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

B1B12 Each column correct

SC B2 use correct matrix from MF1 Can be trig form

(b) (i) (ii)

- B1B12 Stretch, in *x*-direction sf 5
- B1B12 Rotation, 60° clockwise

6

(Q5, June 2010)

35 (i)

$$\det \mathbf{A} = a^2 - a$$

- M1 Show correct expansion process for 3 x 3
- M1 Correct evaluation of any 2 x 2
- A1 3 Obtain correct answer

(ii) (a)

M1 Find a pair of inconsistent equations

(b)

(c)

- A1 State inconsistent or no solutions
-)
- M1 Find a repeated equation A1 State non unique solutions
- A1 B1
 - State that det **A** is non-zero or find correct
 - solution
 - B1 6 State unique solution

SC if detA incorrect, can score 2 marks for correct deduction of a unique

solution, but only once

36 (i)	(7 9)	B1B1 2	Each element correct SC (7,9) scores B1
(ii)	(18)	B1* depB1 2	Obtain correct value Clearly given as a matrix
(iii)	$ \begin{pmatrix} 12 & -4 \\ 6 & -2 \end{pmatrix} $	M1	Obtain 2×2 matrix
		A1 A1 3	Obtain 2 correct elements Obtain other 2 correct elements

(Q1, Jan 2011)

\mathbf{A}^2	B1 (A ⁻¹) ⁻¹ = A seen or implied M1 Use product inverse correctly A1cao 3 Obtain correct answer 3
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(Q5, Jan 2011)

9 (i)	M1 M1	Show correct expansion process for 3 x 3 Correct evaluation of any 2 x 2
$\det_{\mathbf{(ii)}} \mathbf{M}_{3} = a^2 - 7a + 6$ $0 1$	A1 3 B1-B1-2 M1	correct answer Each column correct Solve det M = 0
(ii) $a = 1 \text{ or } 6$	A1A1 3	Phtaigeners begins the singly axes
(iii)	depB1 2	Scale factor $\sqrt{3}$ Attempt to eliminate one variable
(iii) (a)	A1 A1 3 B1 3	(இந்தி), (க்.இ)eindiquations in 2 unknowns குத்நிதத்திற்க number of solutions கேவீக்க்கோழ்கள் ஒருப்புள்ள உருப்புள்ள consistent with their (i) or (ii)
(b) $\det \mathbf{C} = 4$	B1 2 1 2	Correct value found Scale factor for area
	9	(Q9, Jan 20

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40 (i) $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$	$\begin{pmatrix} 4 & 4a\alpha & \beta \\ 2 & 0 \end{pmatrix}$	B1		3 B seen or implied
		B1 B1		Wele hents correct, a.e.f., including brackets
(ii) (4	$\begin{pmatrix} 1 + 4a & 3a \\ 4 & 1 \end{pmatrix}$	M1		Sensible attempt at matrix multiplication
		A1	2	for AB or BA Obtain correct answer
		5		

(Q1, June 2011)

41 $k^2 - 16$	B1 M1		Obtain correct det Equate their det to 0
$k = \pm 4$	A1 3	3	Obtain correct answers

(Q3, June 2011)

2	Show correct expansion process for 3×3 or multiplication of \mathbb{C} and adj \mathbb{C}
	M1 Correct evaluation of any 2×2
$\det \mathbf{C} = \Delta = 5a - 5$	A1 Obtain correct answer
(5 -4 1)	M1 Show correct process for adjoint entries
$ \frac{1}{\Delta} \begin{pmatrix} 5 & -4 & 1 \\ -5 & 4a & -a \\ 5 & -3a-1 & 2a-1 \end{pmatrix} $	A1 Obtain at least 4 correct entries in adjoin
,	A1 Obtain completely correct adjoint
	B1 Divide their adjoint by their determinant
	7

(Q6, June 2011)

43 (i)	B1 B1 B1 3		(0, 3) seen (3, 0) seen Square with A'B' and C' positioned correctly
(ii) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ or $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1*		Reflection in $y = x$ or $y = -x$
	DB1		Correct matrix, dep on stating reflection
$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$ or $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$	B1*		Enlargement scale factor 3 or s.f3
	DB1 4	4	Correct matrix, dep on stating enlargement S.C. B2 for a pair of transformations consistent with their diagram.
	7		

(Q8, June 2011)

44		Б.1	State identity matrix is $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	
		B1 M1	Find a pair of simultaneous equations	
	3p + 4q = 1, $-3p - 5q = 1$, $2p + 3q = 0$	A1 M1	Correct pair of distinct equations Attempt to solve	
	p=3 and $q=-2$	A1 [5]	Obtain correct answers	

(Q2, Jan 2012)

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45	(a)		$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1 B1 [2]	Each column correct	
	(b)	(i)		B1 DB1 [2]	Stretch Scale factor 4 in the y direction	Not "in the y-axis"
	(b)	(ii)	4	B1 B1 [2]	Correct value of determinant Scale factor for area	Allow scale factor of stretch or eqiv.

(Q5, Jan 2012)

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46	(i)	$\det \mathbf{X} = \Delta = 10 - 9a - a^2$	M1 M1 A1 [3]	Show correct expansion process for 3×3 Correct evaluation of any 2×2 Obtain correct answer aef	
	(ii)	a = 1 or -10	M1 A1FT A1FT [3]	Their det $\mathbf{X} = 0$ Obtain correct answers from their (i)	
	(iii)	$ \frac{1}{\Delta} \begin{pmatrix} -a & 2 & 6-9a \\ 5 & -a-9 & 18-3a \\ -a & 2 & a^2-4 \end{pmatrix} $	M1 A1 A1 B1ft	Show correct process for adjoint entries Obtain at least four correct entries in adjoint Obtain completely correct adjoint Divide by their determinant	

 $\alpha + \beta + \gamma$ (Q9, Jan 2012) $\alpha\beta + \beta\gamma \quad \gamma\alpha$

47	(i)	(5 2)	M1	Multiplication attempt, 2 elements correct	
		(13 6)	A1	All elements correct	
			[2]		
	(ii)	EITHER			
		$\mathbf{B}^{-1}\mathbf{A}^{-1} = (\mathbf{A}\mathbf{B})^{-1}$	B1	Stated or used	
			B1ft	Divide by correct determinant	
		1(6 -2)	B1ft	Both diagonals correct	
		$\begin{bmatrix} \frac{1}{4} \begin{pmatrix} 6 & -2 \\ -13 & 5 \end{pmatrix} \end{bmatrix}$			
			[3]		
		OR	B1	Either inverse correct	
			B1	Two elements correct in final answer, both	
				inverses must be correct	
			B1	All elements correct	

(Q2, June 2012)

РМТ

PMT

	depB1	eg image of $(0, 1)$ is $(2, 1)$ or parallel to the <i>x</i> -axis
		owie
		axis
	[2]	
Either		State $\mathbf{Z} = \mathbf{Y}\mathbf{X}$
		Obtain $\mathbf{Y} = \mathbf{Z}\mathbf{X}^{-1}$
$\begin{pmatrix} 1 & -2 \end{pmatrix}$	B1	State or use correct inverse
	M1	Matrix multiplication, 2 elements correct
$\begin{pmatrix} 1 & \sqrt{3} \end{pmatrix}$	A1	Obtain completely correct simplified exact
$\left \begin{array}{ccc} \frac{1}{2} & \frac{1}{2} \end{array} \right $		matrix
$\begin{vmatrix} 1 & \sqrt{3} & 1 \end{vmatrix}$		
$\left(-\frac{\sqrt{2}}{2}, \frac{1}{2}\right)$		
	[5]	
Or	[0]	
z = (a b)(1 2)	B1	Correct order for matrix multiplication
(c. 2010)		
(a 2a+0)	B1	Obtain 2correct elements
	B1	Obtain other 2 correct elements
$\begin{pmatrix} 1 & \sqrt{3} \end{pmatrix}$	3.61	E-mate alamenta 2 account
$\left \begin{array}{ccc} \frac{1}{2} & \frac{1}{2} \end{array} \right $		Equate elements, 2 correct Obtain completely correct simplified exact
	AI	matrix
$\left[\begin{array}{cc} \left[-\frac{\sqrt{2}}{2} & \frac{1}{2}\right] \end{array}\right]$		
	B1*	Rotation
		60° clockwise
	$\begin{pmatrix} 1 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix}$ $\begin{pmatrix} \text{Or} \\ \mathbf{Z} = \begin{pmatrix} a & b \\ c & a \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ $\begin{pmatrix} a & 2a + b \\ c & 2c + a \end{pmatrix}$ $\begin{pmatrix} a & 2a + b \\ c & 2c + a \end{pmatrix}$ $\begin{pmatrix} a & b \\ b & 1 \end{pmatrix}$ $\begin{pmatrix} a & 2a + b \\ c & 2c + a \end{pmatrix}$ $B1$ $B1$ $B1$

(Q9, June 2012)

49	(i)			M1	Show correct expansion process for 3×3	
				M1	Correct evaluation of any 2×2	
			$a^{3} - 4a$	A1	Obtain correct answer	
				[3]		
	(ii)	(a)		В1	det D = 15 so unique sol'n or solve to find correct solution (-2/5, 1, 4/5)	SC B1 once if unique solution following their incorrect det D non zero
				[1]		
	(ii)	(b)		B1	Their det $\mathbf{D} = 0$, so non-unique solutions	
				M1	Attempt to solve equations with $a = 2$	
				A1	Explain inconsistency with correct working	
				[3]		
	(ii)	(c)		B1	Their det $\mathbf{D} = 0$, so non-unique solutions	
				M1	Attempt to solve equations with $a = 0$	
				A1	Explain consistency with correct working	
				[3]		

(Q10, June 2012)

50	(i)	$\begin{pmatrix} 2a-3 & 2 \\ 2 & 5 \end{pmatrix}$	B1	I or 3I seen or used
			B1	2 elements correct
			B1	Other 2 elements correct
			[3]	
	(ii)	$\frac{1}{4a-1}\begin{pmatrix} 4 & -1 \\ -1 & a \end{pmatrix}$ or equivalent	B1	Divide by correct determinant
			B1	Both diagonals correct
			[2]	-

(Q1, Jan 2013)

РМТ

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51	$+$ $3\lambda^2 - 7\lambda + 2$	M1 M1 A1	Show correct expansion process for correct 3 x 3 Correct evaluation of any 2 x 2 Obtain correct 3 term quadratic
	$\frac{1}{3}$ or 2	B1* DM1 A1 [6]	Equate their det to 0 Attempt to solve a quadratic equation Obtain correct answers

(Q5, Jan 2013)

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52	(i)	(1 2)		B1 B1	Each column correct
		(0 2)		[2]	
	(ii)	Either	Or		Either Or
		$(1 \ 0)$	$\begin{pmatrix} 1 & 2 \end{pmatrix}$	B1 DB1	Stretch, s.f. 2 in y direction Shear, x-axis invariant e.g. $(0,1) \rightarrow (2,1)$
		$P:\begin{pmatrix}1&0\\0&2\end{pmatrix}$	$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$	B1	Correct matrix
		(1 1)	$\begin{pmatrix} 1 & 0 \end{pmatrix}$	B1 DB1	Shear, x axis invariant e.g. $(0, 1) \rightarrow (1, 1)$ Stretch, s.f.2 in y direction,
		$Q:\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$	B1	Correct matrix
			` '	[6]	N.B. "in the x/y axis" is incorrect
	(iii)	po.(1 1)	(1 4)	M1	Attempt at matrix multiplication of two 2 x 2 matrices from (ii)
		$PQ: \begin{pmatrix} 1 & 1 \\ 0 & 2 \end{pmatrix}$	$\begin{pmatrix} 1 & 4 \\ 0 & 2 \end{pmatrix}$	A1	Obtain correct result cao
				[2]	

(Q6, Jan 2013)

 $\sqrt{}$

53	(i)	(7 23)	B1B1	Each element correct, missing brackets B1 only
			[2]	
	(ii)	(6 -15)	M1	Obtain 2 × 2 matrix
		$\begin{pmatrix} 4 & -10 \end{pmatrix}$	A1	Obtain 2 correct elements
		(. 10)	A1	Obtain other 2 correct elements
		$\det \mathbf{CB} = 0$	A1FT	Obtain their det CB , must be a 2×2 matrix
		singular	A1FT	Correct conclusion from their det CB
			[5]	

(Q2, June 2013)

54	(i)	$\begin{pmatrix} 0 & 1 \\ \uparrow^1 & 0 \end{pmatrix}$	B1B1	Each column correct
			[2]	
	(ii)	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	B1B1	Each column correct
			[2]	
	(iii)	(0 1)	M1	Attempt at matrix multiplication in correct order
		$\begin{pmatrix} 1 & 0 \end{pmatrix}$	A1FT	Obtain correct answer from their (i) and (ii)
			[2]	
	(iv)	Reflection, in $y = x$	B1B1	Correct description of their (iii) only
			[2]	

(Q7, June 2013)

РМТ

55	(i)		M1	Show correct expansion process for 3×3
			M1	Correct evaluation of any 2×2
		a+3	A1	Obtain correct answer
			M1	Use $\det \mathbf{A} = 0$
		a = -3	A1FT	Obtain correct answer from their det A
			[5]	
	(ii)	(1 -1 1)	M1	Show correct processes for adjoint entries
		$\frac{1}{a+3}$ 7 $a-4$ 1-2a	A1	Obtain at least 4 correct entries in adjoint
		$\begin{bmatrix} \frac{1}{a+3} & 7 & a-4 & 1-2a \\ -11 & 8-a & 3a-2 \end{bmatrix}$	A1	Obtain completely correct adjoint
		(11 0 0 30 2)	B1	Divide adjoint by their det A
		(2)		
		$\left \begin{array}{c} 1 \\ -1 \end{array} \right 2-4a$	M1	Pre-multiply column matrix by their A ⁻¹
		a+3 $7a-1$	A2	Obtain correct answer, A1 for 1 element correct
			[7]	

(Q10, June 2013)